Problem 1 (80%) – Coolant selection for an advanced high-temperature reactor

i) To calculate the mass flow rate, \( \dot{m} \), we use the continuity equation:

\[
\dot{m} = \rho V \left( \frac{\pi}{4} D^2 \right)
\]

where \( \rho \), \( V \) and \( D \) are the coolant density, coolant velocity and channel diameter, respectively. The friction pressure drop in the coolant channel, \( \Delta P_{\text{fric}} \), can be calculated as:

\[
\Delta P_{\text{fric}} = f \cdot \frac{L}{D} \cdot \rho V^2
\]

where \( f \) and \( L \), are the friction factor and channel length, respectively. The generic expression of the friction factor for fully-developed flow in smooth channels is:

\[
f = \frac{C}{Re^n}
\]

where \( Re \) is the Reynolds number, \( Re = \frac{\rho V D}{\mu} \), and \( C \) and \( n \) are numerical coefficients depending on the flow regime. Substituting Eq. (3) into Eq. (2), we get:

\[
\Delta P_{\text{fric}} = \frac{C}{Re^n} \cdot \frac{L}{D} \cdot \rho V^2 = \frac{C}{2} \cdot \frac{\mu L \rho^{1-n} V^{2-n}}{D^{1+n}}
\]

Since \( \Delta P_{\text{fric}} \) is given, Eq. (4) can be solved to find the coolant velocity:

\[
V = \left( \frac{2 \Delta P_{\text{fric}} D^{1+n}}{C \mu L \rho^{1-n}} \right)^{1/(2-n)}
\]

Because the flow regime is not known a priori, one has to guess it and then verify the accuracy of the guess. Let us assume turbulent flow with \( Re > 30,000 \) for which \( C = 0.184 \), \( n = 0.2 \) (see Eq. 9-79 in the textbook). Then, Eq. (5) yields \( V = 5.82 \) m/s for liquid sodium and \( V = 2.95 \) m/s for the liquid salt. The corresponding Reynolds numbers are 267,250 and 28,656, respectively. So the flow regime is indeed turbulent, but it is not high-Re turbulent for the liquid salt, because \( Re < 30,000 \). So, Eq. (5) has to be recomputed for the liquid salt with \( C = 0.316 \) and \( n = 0.25 \) (see Eq. 9-80 in the textbook), which gives \( V = 2.91 \) m/s and \( Re = 28,207 \).
The corresponding mass flow rates are calculated from Eq. (1) and are 0.357 kg/s and 0.443 kg/s for liquid sodium and liquid salt, respectively.

ii) The pumping power due to friction, \( \dot{W}_p \), can be calculated as follows:

\[
\dot{W}_p = \dot{m} \cdot \frac{\Delta P_{\text{fric}}}{\rho}
\]  

(6)

Thus, the pumping power is about 91 W and 46 W for liquid sodium and liquid salt, respectively.

iii) Since the power profile is axially uniform, the maximum temperature in the fuel occurs at the channel outlet (\( z=L \)). Per the hint, let us approximate the fuel around the coolant channel as an annulus of inner diameter \( D \) and outer diameter \( D_f \), where \( D_f \) is calculated imposing the conservation of the fuel volume:

\[
\left( \frac{\sqrt{3}}{2} w^2 - \frac{\pi}{4} D^2 \right) \cdot L = \left( \frac{\pi}{4} D_f^2 - \frac{\pi}{4} D^2 \right) \cdot L \\
\Rightarrow D_f = \sqrt{\frac{2\sqrt{3}}{\pi}} w = 3.15 \text{ cm}
\]  

(7)

where \( w = 3 \text{ cm} \) is the width of the hexagonal unit cell of the core (see Figure 1).

![Figure 1. Equivalent annulus.](image)

The heat conduction equation in this annulus is:

\[
\frac{1}{r} \frac{d}{dr} \left[ k_f r \frac{dT}{dr} \right] + q'''' = 0
\]  

(8)

where \( r \) is the radial coordinate, \( k_f \) is the fuel thermal conductivity (=6 W/m·K, independent of temperature) and \( q'''' \) is the volumetric heat generation rate within the fuel. The linear power, \( q' \), is related to \( q'''' \) as follows:

\[
q' = \frac{\pi}{4} \left( D_f^2 - D^2 \right) q''''
\]  

(9)

The boundary condition for Eq. (8) is:

\[-k_f \frac{dT}{dr} = 0 \quad \text{at} \ r = D_f/2\]  

(10)
The solution of Eq. (8) is then:

$$\therefore T_{\text{max}} - T_{s,L} = \frac{q'}{2\pi k_f} \left[ \frac{D_f^2}{D_f^2 - D^2} \ln \left( \frac{D_f}{D} \right) - \frac{1}{2} \right]$$  \hspace{1cm} (11)$$

where $T_{\text{max}} = 1000^\circ\text{C}$ is the maximum temperature in the fuel, $T_{s,L}$ is the (unknown) temperature at the surface of the coolant channel at $z=L$. Newton's law of cooling provides a relationship between $T_{s,L}$ and $T_{b,L}$, i.e., the coolant bulk temperature at the channel outlet:

$$\frac{q'}{\pi D} = h(T_{s,L} - T_{b,L}) \quad \Rightarrow \quad T_{s,L} = T_{b,L} + \frac{q'}{\pi Dh}$$  \hspace{1cm} (12)$$

where $h$ is the heat transfer coefficient, which can be calculated from Eq. 10-113 in the textbook for the liquid sodium ($Pr=0.037$, $Re=267,250 \Rightarrow Nu=13.2$), and the Dittus-Boelter correlation for the liquid salt ($Pr=4.82$, $Re=28,207 \Rightarrow Nu=156.7$). The values of ‘$h$’ are then 79.2 kW/m$^2$K and 15.7 kW/m$^2$K for liquid sodium and liquid salt, respectively.

The bulk temperature at the channel outlet can be calculated from the energy equation as follows:

$$\dot{m}c_p(T_{b,L} - T_{b,o}) = q'L \quad \Rightarrow \quad T_{b,L} = T_{b,o} + \frac{q'L}{\dot{m}c_p}$$  \hspace{1cm} (13)$$

Where $T_{b,o}=600^\circ\text{C}$ is the inlet temperature. Combining Eq. (11), (12) and (13), one gets:

$$T_{\text{max}} - T_{b,o} = \frac{q'}{2\pi k_f} \left[ \frac{D_f^2}{D_f^2 - D^2} \ln \left( \frac{D_f}{D} \right) - \frac{1}{2} \right] + \frac{q'}{\pi Dh} + \frac{q'L}{\dot{m}c_p}$$  \hspace{1cm} (14)$$

which can be solved for $q'$:

$$q' = \frac{T_{\text{max}} - T_{b,o}}{\frac{1}{2\pi k_f} \left[ \frac{D_f^2}{D_f^2 - D^2} \ln \left( \frac{D_f}{D} \right) - \frac{1}{2} \right] + \frac{1}{\pi Dh} + \frac{L}{\dot{m}c_p}}$$  \hspace{1cm} (15)$$

Equation (15) gives $q'=9.4$ kW/m and $q'=12.5$ kW/m for liquid sodium and liquid salt, respectively.

iv) The thermal-hydraulic analysis indicates that a liquid salt coolant is the better choice, as it affords higher heat removal rates while requiring lower pumping power. The good thermal-hydraulic performance of the liquid salt is due mainly to its high heat capacity ($\dot{m}c_p$).
Problem 2 (20%) – Flow split in downflow

Because the two channels are connected to the same inlet and outlet plena, the total pressure change in each channel, $-\Delta P_{\text{tot}}$, is the same:

\[-\Delta P_{\text{tot}} = f \cdot \frac{L}{D_e} \cdot \frac{G_1^2}{2 \rho_1} - \rho_1 g L \tag{16}\]
\[-\Delta P_{\text{tot}} = f \cdot \frac{L}{D_e} \cdot \frac{G_2^2}{2 \rho_2} - \rho_2 g L \tag{17}\]

where the form and acceleration terms were neglected, $f$ is the friction factor (assumed equal in both channels, as per the problem statement), $D_e$ is the hydraulic diameter of the channels, $G_1$ and $G_2$ are the mass fluxes in channel 1 and 2, respectively, and $\rho_1$ and $\rho_2$ are the average water densities in channel 1 and 2, respectively. Eliminating $-\Delta P_{\text{tot}}$ from Eq. (16) and (17) and recognizing that $\rho_1 > \rho_2$ (i.e., channel 1 is cooled, while channel 2 is heated), one gets:

\[\frac{G_1^2}{\rho_1} > \frac{G_2^2}{\rho_2} \quad \Rightarrow \quad G_1 > \sqrt{\frac{\rho_1}{\rho_2}} G_2 > G_2 \tag{18}\]

Therefore, the mass flow rate in channel 1 is higher than the mass flow rate in channel 2. This result is also intuitive because cooling channel 1 and heating channel 2 creates a “chimney” effect that opposes downflow in channel 2.