14.123 Microeconomic Theory III
Spring 2009

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1. Correlated Equilibrium
2. The Power of Correlation
3. Review

Read: FT 2.2, everything (including Pb sets & solutions)
A Coordination Game

• Three Nash equilibria: (U,L), (L,R), and “equal-probability mixed”.

• Nash equilibrium can be thought of as a self-enforcing agreement.

• Why not use a more sophisticated agreement, like this:
  – Ask a third party (or computer, iPhone, etc.) to generate a random $x = 1, 2$ or $3$ with equal probs; players don’t learn $x$.
  – If $x = 1$ then 3rd party tells P1 to play $U$, P2 to play $L$.
    If $x = 2$ then 3rd party tells P1 to play $D$, P2 to play $L$.
    If $x = 3$ then 3rd party tells P1 to play $D$, P2 to play $R$.

• Joint randomization device.

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>5,1</td>
<td>0,0</td>
</tr>
<tr>
<td>D</td>
<td>4,4</td>
<td>1,5</td>
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Public Randomization Helps

- Proposition: In the two-stage game where first the 3rd party generates $x = 1, 2, 3$ with equal probabilities and then tells players which action to choose according to the rules specified above, it is PBE for P1 and P2 to follow the 3rd party’s recommendations.

- When P1 is told “play $U$”, he believes P2 plays $L$, so $U$ is optimal. When P1 is told “play $D$”, he believes P2 plays $L$ and $R$ with 50-50% chance, so $U$ and $D$ are equally good, may as well play $D$. Symmetric argument works for P2.

- Payoff $(10/3, 10/3)$ is outside the convex hull of NE payoffs!
Correlated Equilibrium

• **DEF:** A joint distribution over strategy-profiles, \( q \in \Delta(\times_i S_i) \) is a correlated equilibrium if for every \( i \) and \( s_i \in S_i \),

\[
s_i \in \arg\max_{s_i' \in S_i} \sum_{s_{-i}} q(s_i, s_{-i}) u_i(s_i', s_{-i}).
\]

• **Interpretation:** We agree on the joint distribution of strategies, \( q \). The 3\(^{rd} \) party / machine tells you to play \( s_i \); you know the other players use \( s_{-i} \) with probability proportional to \( q(s_i, s_{-i}) \). Your expected payoff given this information is maximized by \( s_i' = s_i \).

• Note that a CE is just a distribution over outcomes (entries in the matrix representing the normal form game) that satisfies a set of linear inequalities.

• **THM:** Every NE is CE. Convex combo of CE is also CE.
Discussion

• Correlated equilibrium is like an “incentive compatible mechanism”. Think of this when you’ll study mechanism design.

• Some maintain (e.g., Robert Aumann) that CE is a natural solution concept for rational players with a common prior.

  Key idea: Players have not only a common prior about payoff-relevant random parameters, but also about each others’ strategies.

  Knowing what $i$ is supposed to play ($s_i$) and the joint distribution of ($s_i, s_{-i}$), a rational player $i$ adheres to $s_i$, which is the def. of CE.

• Others may say (e.g., Eric Maskin) that moves that facilitate joint randomization (pre-play communication, mechanism) should be directly modeled in the game, not in the equilibrium concept.
2. The Power of Correlation

- Somewhat related is the following mechanism-design idea: Myerson’s side-bet auction with correlated valuations.
- 2 buyers, 2 possible valuations for each: $\theta_i \in \{10, 100\}, i=1,2$.
- $\Pr(\theta_1 = \theta_2 = 10) = \Pr(\theta_1 = \theta_2 = 100) = 1/3$; $\Pr(\theta_i = 10, \theta_{-i} = 100) = 1/6$.
- **Auction**: Buyers are asked to bid either 10 or 100. Highest bid wins (50-50% tie-break), winner pays own bid. In addition, if $i$ reports $\theta_i = 10$ then he gets 15 if $\theta_{-i} = 10$, but pays 30 if $\theta_{-i} = 100$. (Called a side-bet auction because of the last twist.)
Proposition

Truthful bidding is zero-profit equilibrium for the buyers.

1) If \( \theta_i \) reports truthfully then he gets zero payoff.

2) If \( \theta_i = 10 \) then bidding 100 is not profitable, has to pay 100 if wins.

3) If \( \theta_i = 100 \) reports 10 then \( i \) may win only if other reports \( \theta_{-i} = 10 \). 

\[
\text{Payoff} = \frac{2}{3}(-30) + \frac{1}{3}(15 + \frac{1}{2}(100-10)) = 0.
\]

1) Imperfect correlation of valuations allows First Best for seller.

2) This is a striking result. It shows how powerful correlated information is in a game where all parties are risk neutral.
3. Review: Part I, Decision Theory

- Model with objective, reduced lotteries
  - Expected Utility Axioms & Theorem
  - Graphical analysis with 3 outcomes (in the simplex / triangle)
  - Allais paradox, framing effects

- Money lotteries and EU analysis
  - Risk aversion and the concavity of vNM utility fcn
  - Measuring risk aversion: \( r_A(w,u) \), risk premium, certainty equivalent, relationship among these concepts.
  - Insurance and Optimal Portfolio Choice applications
  - DARA, CARA, work with concrete functional form
  - Prudence and precautionary premium; relationship to DARA.
Review of Decision Theory, cont.

- Money lotteries, continued:
  - Relative risk aversion; CRRA, log-utility
  - Stochastic orders: FSD, SSD, MPR, MLR
  - Comparative statics with respect to changes in risk aversion and changes in the riskiness of the gamble
  - Background risk

- Other models, other theories
  - State-dependent EU (just definition, possible uses)
  - Subjective EU (difference from EU over objective lotteries)
  - Ellsberg’s paradox.
Review: Part II, Game Theory

- Solution concepts (general games):
  - Iterated Strong Dominance
  - Rationalizability
  - Correlated Equilibrium
  - Nash Equilibrium
  - Subgame Perfection
  - Sequential Equilibrium
  - Trembling-Hand Perfection (in agent-normal form)
  - Stability

- Solution concepts (signaling games):
  - PBE
  - Intuitive Criterion, D1
Review of Game Theory, cont.

• Techniques:
  – Backward induction, iterated elimination of strictly dominated strategies, iterated conditional dominance.
  – Single-deviation principle.
  – Checking sequential equilibrium, trembling-hand perfection, intuitive criterion, D1, Stability.
  – Global games perturbation and selection technique.
  – Finding / checking bayesian equilibria in games (e.g., auctions, other games of incomplete information).
  – Proofs with “single-crossing” arguments (e.g., revenue comparisons in auctions).
Review of Game Theory, cont.

• Dynamic games:
  – Bargaining models (alternating-offers bargaining, finding unique SPE in it)
  – Finitely Repeated Games
    Benoit-Krishna Folk Theorem
  – Infinitely Repeated Games
    Fudenberg-Maskin Perfect Folk Theorem
  – Renegotiation Proofness

• Bayesian games:
  – Signaling and communication games
  – Auctions