12.540 Principles of Global Positioning Systems
Spring 2008

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12.540 Principles of the Global Positioning System
Lecture 14

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Propagation Medium

• Propagation:
  – Signal propagation from satellite to receiver
  – Light-time iteration
  – Basic atmospheric and ionospheric delays
  – Propagation near receiving antenna
Propagation

• Basics:
  – Signal, tagged with time from satellite clock, transmitted.
  – About 60 msec (20,000 km) later the signal arrives at GPS receiver. Satellite has moved about 66 m during the time it takes signal to propagate to receiver.
  – Time the signal is received is given by clock in receiver. Difference between transmit time and receive time is pseudorange.
  – During the propagation, signal passes through the ionosphere (10-100 m of delay, phase advance), and neutral atmosphere (2.3-30 m depending on elevation angle).
Propagation

• To determine an accurate position from range data, we need to account for all these propagation effects and time offsets.

• In later lectures, examine ionospheric and atmospheric delays, and effects near antenna.

• Basic clock treatment in GPS
  – True time of reception of signal needed
  – True time of transmission needed (af0, af1 from broadcast ephemeris initially OK)
  – Position of satellite when signal transmitted
Times

- RINEX data files, tag measurements by reception time as given by the receiver clock. The error in the receiver time must be determined iteratively.
- For linearized least squares or Kalman filter need to establish non-linear model and then estimator determines adjustments to parameters of model (e.g. receiver site coordinates) and initial clock error estimates that “best” match the data.
Non-linear model

• Basics of non-linear model:
  – Rinex data file time tags give approximate time measurement was made.
  – Using this time initially, position of satellite can be computed
  – Range computed from receiver and satellite position
  – Difference between observed pseudorange and computed ranges, gives effects of satellite and receiver clock errors. In point positioning, satellite clock error is assumed known and when removed from difference, error in receiver clock determined.
  – With new estimate of receiver clock, process can be iterated.
  – If receiver position poorly known, then whole system can be iterated with updated receiver coordinates.
Sensitivities

• Satellites move at about 1km/sec, therefore an error of 1 msec in time results in 1 m satellite position (and therefore in range estimate and receiver position).
• For pseudo-range positioning, 1 msec errors OK. For phase positioning (1 mm), times needed to 1 μsec.
• (1 μsec is about 300 m of range. Pseudorange accuracy of a few meters in fine).
“Light-time-iteration”

• To compute theoretical range; two basic methods used
  – (a) “Doppler shift corrections” ie. Account for rate of change of range during propagation time
  – (b) “Light-time-iteration” Method most commonly used.

• Light time iteration: Basic process is to compute range using simple Cartesian geometry but with position of receiver at receive time and position of transmitter at transmit time.
Light-time-iteration

• Light time iteration must be computed in a non-rotating frame
• Reason: Consider earth-fixed frame: one would simply compute Earth fixed coordinates at earlier time. In non-rotating frame, rotation to inertial coordinates would be done at two different time (receiver when signal received; transmitted when signal transmitted).
• Difference is rotation of Earth on ~60 msec. Rotation rate ~460 m/sec; therefore difference is about 30 meters.
Clock errors

PRN 03 (June 14)

Clock error (ns)

Time (hrs)
Relativistic effects

• General relativity affects GPS in three ways
  – Equations of motions of satellite
  – Rates at which clock run
  – Signal propagation
• In our GPS analysis we account for the second two items
• Orbits only integrated for 1-3 days and equation of motion term is considered small
Clock effects

- GPS is controlled by 10.23 MHz oscillators
- On the Earth’s surface these oscillators are set to $10.23 \times (1 - 4.4647 \times 10^{-10})$ MHz (39,000 ns/day rate difference)
- This offset accounts for the change in potential and average velocity once the satellite is launched.
- The first GPS satellites had a switch to turn this effect on. They were launched with “Newtonian” clocks
Propagation and clock effects

• Our theoretical delay calculations are made in an Earth centered, non-rotating frame using a “light-time” iteration i.e., the satellite position at transmit time is differenced from ground station position at receive time.

• Two corrections are then applied to this calculation
Corrections terms

- Propagation path curvature due to Earth’s potential (a few centimeters)
  \[ \Delta \tau = \frac{2GM}{c^3} \ln \left( \frac{R_r + R_s + \rho}{R_r + R_s - \rho} \right) \]

- Clock effects due to changing potential
  \[ \Delta \tau = -\sqrt{\frac{GM}{c^2}} e \sqrt{a \sin E} \]

- For \( e=0.02 \) effect is 47 ns (14 m)
Effects of General Relativity

PRN 03 Detrended; e=0.02

- Clock error (ns)
  - Clock - trend (ns)
  - GR Effect (ns)

Time (hrs)

0  4  8  12  16  20  24

0  25  50

-50  -25  0  25  50
Tests of General Relativity

• In the parameterized post-Newtonian formulation, the time delay expression becomes:

\[
\Delta \tau = \frac{-\sqrt{GM}}{c^2} \frac{(1+\gamma)}{2} e^{\sqrt{a} \sin E}
\]

• In PPN, \(\gamma\) is the gravitational term. In general relativity \(\gamma = 1\)

• The clock estimates from each GPS satellite allow daily estimates of \(\gamma\). Interesting project for someone.
Using GPS to determine $\gamma$

- Each day we can fit a linear trend and once-per-revolution sin and cos terms to the each of the 27-28 GPS satellites.
- Comparison between the amplitude and phase (relative to sin($E$)) allows and estimate of gamma to be obtained.
- Quadrature estimates allows error bound to be assessed (cos($E$) term).
- Problem: Once-per-orbit perturbations are common. However should not be proportional to eccentricity.
Examples of receiver clock behavior

• Examples of satellite and station clock behaviors can be found at:
• http://geoweb.mit.edu/~tah/MITClk
• IGS Time Standards are given at: https://goby.nrl.navy.mil/IGStime/index.php
• Directories are by GPS week number and directories ending in W are total clock estimates; folders ending in D are differences between IGS analysis centers
• Now examine some examples
Receiver clocks: ASC1
Receiver Clock: HOB2 Hydrogen Maser
ASC/HOB2 Linear trends removed
HOB2 only

![Graph showing HOB2 detrended (m) over a range of days from 14.0 to 15.5. The graph includes horizontal lines at -0.5, 0.0, 0.5, and 1.0 detrended (m).]
Summary of clocks

• In some cases; clock are well enough behaved that linear polynomials can be used.
• Most commonly: receiver clocks are estimated at every measurement epoch (white noise clocks) or GPS data is differenced to remove clock (as in question 2 of HW 2).
• Errors in receiver clocks are often thousands of km of equivalent time. Homework #3 will show a “bad” clock in receiver.