12.540 Principles of Global Positioning Systems
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12.540 Principles of the Global Positioning System
Lecture 02
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Coordinate Systems

• Today we cover:
  – Definition of coordinates
  – Conventional “realization” of coordinates
  – Modern realizations using spaced based geodetic systems (such as GPS).
Coordinate system definition

• To define a coordinate system you need to define:
  – Its origin (3 component)
  – Its orientation (3 components, usually the direction cosines of one axis and one component of another axes, and definition of handed-ness)
  – Its scale (units)
Coordinate system definition

• In all 7 quantities are needed to uniquely specify the frame.
• In practice these quantities are determined as the relationship between two different frames.
• How do we measure coordinates?
• How do we define the frames?
Measuring coordinates

- Direct measurement (OK for graph paper)
- Triangulation: Snell 1600s: Measure angles of triangles and one-distance in base triangle
- Distance measured with calibrated “chain” or steel band (about 100 meters long)
- “Baseline” was about 1 km long
- Triangles can build from small to larger ones.
- Technique used until 1950s.
Measuring coordinates

- Small errors in the initial length measurement, would scale the whole network.
- Because of the Earth is "nearly" flat, measuring angles in horizontal plane only allows "horizontal coordinates" to be determined.
- Another technique is needed for heights.
Measuring coordinates

• In 1950s, electronic distance measurement (EDM) became available (out growth of radar)
• Used light travel times to measure distance (strictly, travel times of modulation on either radio, light or near-infrared signals)
Measuring coordinates

- Advent of EDM allowed direct measurements of sides of triangles
- Since all distances measured less prone to scale errors.
- However, still only good for horizontal coordinates
Accuracies

- Angles can be measured to about 1 arc second \((5 \times 10^{-6} \text{ radians})\)
- EDM measures distances to \(1 \times 10^{-6}\) (1 part-per-million, ppm)
- Atmospheric refraction 300 ppm
- Atmospheric bending can be 60” (more effect on vertical angles)
Height coordinates

- Two major techniques:
  - Measurement of vertical angles (atmospheric refraction)
  - "Leveling" measurement of height differences over short distances (<50 meters).
  - Level lines were used to transfer height information from one location to another.
Other methods

- Maps were made with “plotting tables” (small telescope and angular distance measurements - angle subtended by a known distance)
- Aerial photogrammetry coordinates inferred from positions in photographs. Method used for most maps
Other methods

• What is latitude and longitude
• Based on spherical model what quantities might be measured
• How does the rotation of the Earth appear when you look at the stars?
• Concept of astronomical coordinates
Geodetic coordinates: Latitude

- Geoid
- Gravity direction
- Normal to ellipsoid
- Earth's surface
- Local equipotential surface
- Latitude, φ
- North
Longitude

Longitude measured by time difference of astronomical events
Astronomical coordinates

• Return to later but on the global scale these provide another method of determining coordinates
• They also involve the Earth’s gravity field
• Enters intrinsically in triangulation and trilateration through the planes angles are measured in
Web sites about geodetic measurements


• [http://www.ngs.noaa.gov/](http://www.ngs.noaa.gov/) is web page of National Geodetic Survey which coordinates national coordinate systems
Earth’s Gravity field

• All gravity fields satisfy Laplace’s equation in free space or material of density $\rho$. If $V$ is the gravitational potential then

\[
\nabla^2 V = 0
\]
\[
\nabla^2 V = 4\pi G \rho
\]
Solution to gravity potential

- The homogeneous form of this equation is a “classic” partial differential equation.
- In spherical coordinates solved by separation of variables, $r=$radius, $\lambda=$longitude and $\theta=$co-latitude

$$V(r, \theta, \lambda) = R(r)g(\theta)h(\lambda)$$
Summary

• Examined conventional methods of measuring coordinates
• Triangulation, trilateration and leveling
• Astronomical positioning uses external bodies and the direction of gravity field
• Continue with the use of the gravity field.